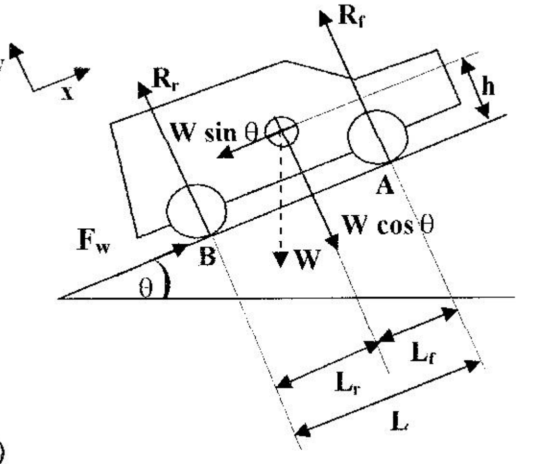
Maximum gradient the car can climb (constant velocity)

I-Adhesion

A- Rear wheel drive (neglect the air resistance, and acceleration):



$\Sigma M_A = 0$

$W\cos\theta\ L_f + W\sin\theta\ h - R_r L = 0$ . . . .(A-1)

$\Sigma F_x = 0$

$F_w = W\sin\theta$

But $(F_w)_{max} = \mu R_r = W\sin\theta$

$$R_r = \frac{w\sin\theta}{\mu} \qquad \text{. . . . .(A-2)}$$

Substituting Eq. (A-1) into Eq. (A-2)

$$W\cos\theta L_f + W\sin\theta\ h - \frac{W\sin\theta}{\mu}L = 0 \qquad \text{. . . . .(A-3)}$$

Divide Eq. (3) by (W cos θ)

$$L_f + \tan\theta\ h - \tan\theta\ \frac{L}{\mu} = 0$$

$$L_f = \tan\theta\ \frac{L}{\mu} - \tan\theta\, h = \tan\theta(\frac{L - h\mu}{L})$$

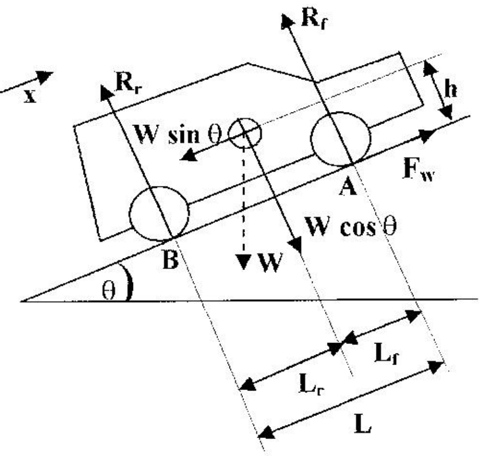
$$\tan\theta = \frac{\mu L_f}{L - \mu h} \qquad \text{. . . . .(A-4)}$$

Taking rolling resistance ($RR = W f_r$) into consideration:

$$\tan\theta = \frac{\mu L_f - f_r}{L - \mu h}$$

$$\theta = \tan^{-1}(\frac{\mu(L_F/L) - f_r}{1 - \mu(h/L)}) \qquad \text{. . . . .(A-5)}$$

B- Front wheel drive (neglect the air resistance, and acceleration):



$\Sigma M_B = 0$

$W \sin\theta \, h - W \cos\theta \, L_r + R_f L = 0$ . . . .(B-1)

$\Sigma F_x = 0$

$F_w = W \sin\theta$

But $(F_w)_{max} = \mu R_f = W \sin\theta$

$$R_f = \frac{w \sin\theta}{\mu}$$ . . . .(B-2)

Substituting Eq. (B-1) into Eq. (B-2)

$$W \sin\theta \, h - W \cos\theta \, L_r + \frac{W \sin\theta}{\mu} L = 0$$ . . . .(B-3)

Divide Eq. (3) by (W cosθ)

$$\tan\theta \, h - L_r + \tan\theta \, \frac{L}{\mu} = 0$$

$$L_r = \tan\theta \, \frac{L}{\mu} + \tan\theta \, h = \tan\theta \left(\frac{L + h\mu}{L}\right)$$

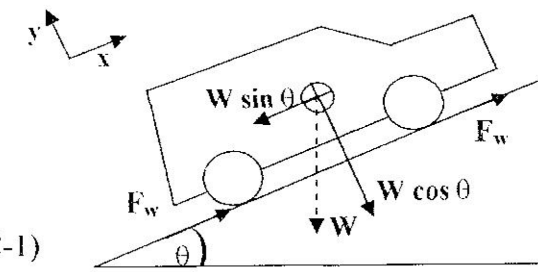
$$\tan\theta = \frac{\mu L_r}{L + \mu h}$$ . . . .(B-4)

Taking rolling resistance ($RR = W f_r$) into consideration:

$$\tan\theta = \frac{\mu L_r - f_r}{L + \mu h}$$

$$\theta = \tan^{-1}\left(\frac{\mu (L_r/L) - f_r}{1 + \mu (h/L)}\right)$$ . . . .(B-5)

C- Four wheel drive (neglect the air resistance, and acceleration):



$\Sigma F_x = 0$

$F_w = W \sin\theta$

But $(F_w)_{max} = \mu W \cos\theta = W \sin\theta$

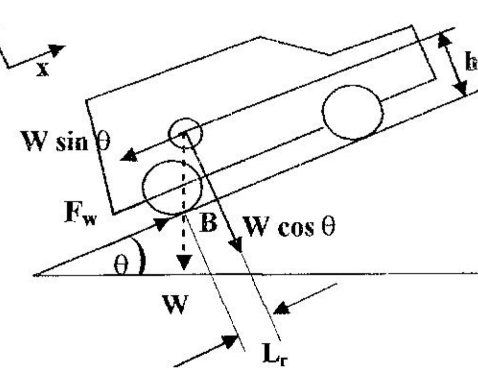
$\tan\theta = \mu$ . . . . (C-1)

Taking rolling resistance ($RR = W f_r$) into consideration:

$\tan\theta = \mu - f_r$ . . . . (C-2)

II- Overturn ($R_f = 0$)

a- Constant speed ($F_i = 0$):



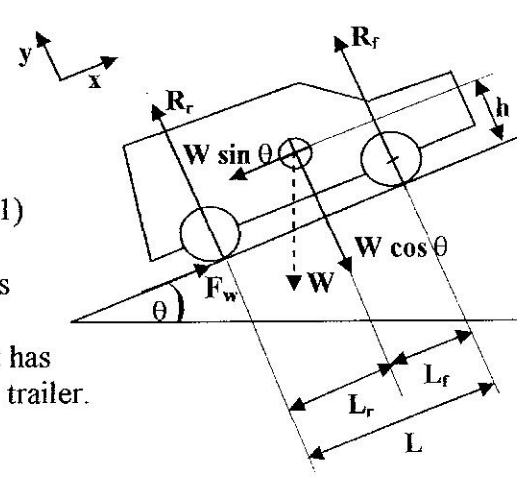
$\Sigma M_B = 0$

$W \sin\theta\, h - W \cos\theta\, L_r = 0$ . . . . (II-1)

* where $R_f = 0$

$$\tan\theta = \frac{L_r}{h} = \frac{L_r/L}{h/L}$$

b- Acclerated car ($F_i = M a$):
The worst condition happens at the first gearbox shift, at maximum acceleration. $F_i h = T_w$



$\Sigma M_B = 0$
$T_{w\,max}\ i_g\ i_f + W \sin\theta\, h - W \cos\theta\, L_r = 0$
where $R_f = 0$
Overturn condition:
$T_{max}\ i_g\ i_f > W (L_r \cos\theta - h \sin\theta)$ . . . . (II-1)

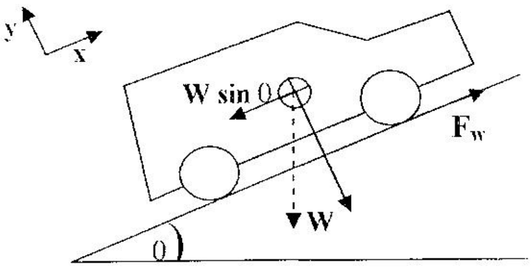
* It is mostly happened to the beach buggy where it has a rear engine (small L_r and L)
* It is also happened to the agricultural tractor where it has a high center of gravity and in case it is coupled to a trailer.

III- Tractive effort

$\Sigma F_x = 0$

$F_w - W \sin\theta = 0$ ....(III-1)

$\sin\theta = \frac{F_w}{W}$ ....(III-2)



* where $F_w < \mu W$

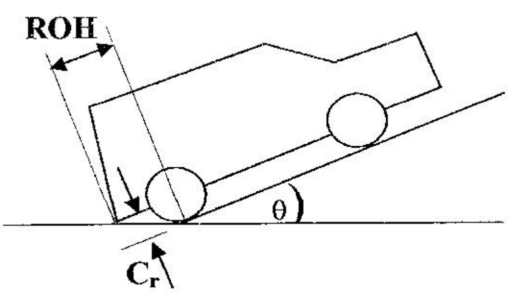
Taking rolling resistance ($RR = W f_r$) into consideration:

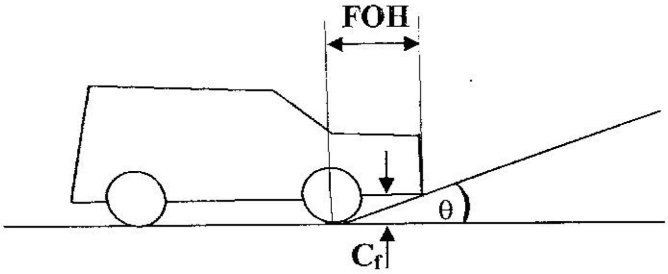
$$\sin\theta = \frac{F_w - W f_r}{W}$$ ....(III-3)

IV- Car dimensions

$$\tan\theta = \frac{C_f}{FOH}, \text{ or}$$ ....(IV-1)

$$\tan\theta = \frac{C_r}{ROH}$$ ....(IV-2)





where

FOH = front over hanging
C_f = front under body clearance
ROH = rear over hanging
C_r = rear under body clearance

Example:

A car has the following data:

$m = 1200$ kg	$i_g = (3.2, 2.8, 1.6, 1)$	$R_w = 30$ mm	FOH = 1.3 m
$T_{max} = 180$ N.m	$i_f = 3.8$	$C_f = C_r = 0.35$ m	ROH = 1.0 m
$L = 2.6$ m	$L_f = 1.0$ m	$L_r = 1.6$ m	$h = 0.6$ m

and

$\mu = 0.8$	$f_r = 0.015$

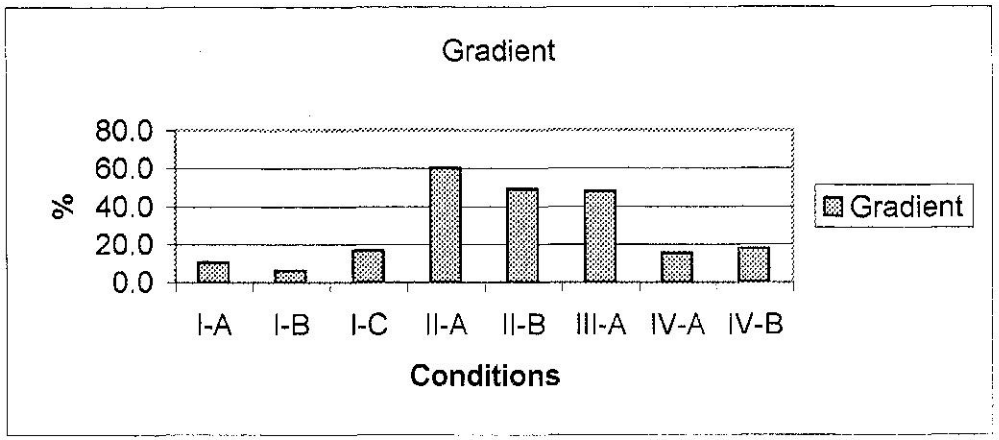
Find the maximum gradient the car can climb at all possibilities.

m	1200	L	2.6	FOH	1.3	Fw	8917.333
igr1	3.2	Lf	1.56	ROH	1.1	W	11772
if	3.8	Lr	1.04	Mu	0.3		
Rw	0.3	h	0.6	fr	0.015	PI	3.141593
T	220	Cfr	0.35				

1	2	3	4	5	6	7	8
I-A	I-B	I-C	II-A	II-B	III-A	IV-A	IV-B
10.6	6.1	16.7	60.0	49.0	47.9	15.1	17.7

Th	40	42	44	46	48	50	52	54	56
D	-2163.25	-1696.83	-1225.08	-748.578	-267.905	216.3535	703.608	1193.265	1684.727

D = T igr1 if - W Lr cosθ +W (h-Rw) sinθ



Inclined Road Forces

Gradient Resistance (GR)

The gradient resistance (climbing resistance, inclined road force) depends on the angle of the road inclination and the weight of the car.

$$GR = W \sin\theta = mg \sin\theta$$

where:

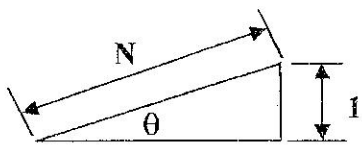
W= the car weight (N) = mg

θ = the angle of road inclination

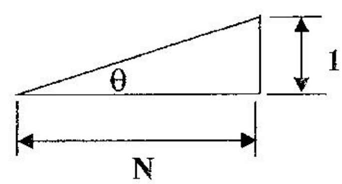
Road inclination (gradient):

The road gradient can be described as 1 in N, this description can be either:

a-



b-



The description (b) is not suitable incase of level road the gradient will be1 in ∞, especially when using the computer. So, the description of the gradient will be as in (a), and the gradient will be ($G = \sin\theta$). The gradient can be written in as a percentage ($G = \sin\theta = S\%$).

* For small angle (θ (radians) $\cong \sin\theta \cong \tan\theta$)
* the road gradient in the highway usually does not exceed 4% and on the local roads it could reach 10%-12%.
* the steepest gradient the vehicle is expected to climb (this may normally be taken as 20%, that is 1 in 5)

The gradient effect on the car:

Up hill:

1- Increase the car motion resistance; $F_G = W \sin\theta$ (against the direction of motion)
2- Increase the load on rear axle and decrease the load on the front one.
3- Decrease the stopping distance when using the brakes (4% is equal to 0.04 g)

Down hill:

1- Increase the tractive effort; $F_G = W \sin\theta$ (in the direction of motion)
2- Increase the load on front axle and decrease the load on the rear one.
3- Increase the stopping distance when using the brakes (4% is equal to 0.04 g)