

## BRAKING PERFORMANCE

Braking performance of motor vehicles is undoubtedly one of the most important characteristics that affect vehicle safety. With increasing emphasis on traffic safety in recent years, intensive efforts have been directed toward improving the braking performance. Safety standards that specify performance requirements of various types of brake system have been introduced in many countries.

### Braking Characteristics of a Two-Axle Vehicle

The major external forces acting on a decelerating two-axle vehicle are shown in the Figure. The braking force  $F$ , originating from the brake system and developed on the tire-road interface is the primary retarding force.

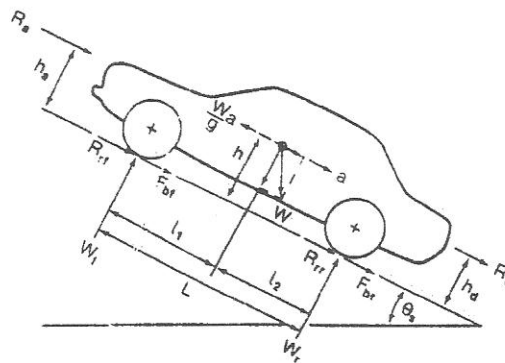


Fig. Forces acting on a two-axle vehicle during braking.

braking force  $F_b$  is given by

$$F_b = (M_b - \sum I \alpha_{an}) / r$$

where  $M_b$  is the applied brake torque,  $I$  is the rotating inertia being decelerated,  $\alpha_{an}$  is the corresponding angular deceleration, and  $r$  is the rolling radius of the tire.

In addition to the braking force, the rolling resistance of tires, aerodynamic resistance, transmission resistance, and grade resistance (when traveling on a slope) also affect vehicle motion during braking.

Thus the resultant retarding force  $F_{res}$  can be expressed by

$$F_{res} = F_b + f_r W \cos \theta_s + R_a \pm W \sin \theta_s + R_t \quad (1)$$

where  $f_r$  is the rolling resistance coefficient,  $W$  is the vehicle weight,  $\theta_s$  is the angle of the slope with the horizontal,  $R_a$  is the aerodynamic resistance, and  $R_t$  is the transmission resistance. When the vehicle is moving uphill, the positive sign for the term  $W \sin \theta_s$  should be used. On a downhill grade, the negative sign should, however, be used. Normally, the magnitude of the transmission resistance is small and can be neglected in braking performance calculations.

During braking, there is load transfer from the rear axle to the front axle. By considering the equilibrium of moments about the front and rear tire-ground contact points, the normal loads on the front and rear axles  $W_f$  and  $W_r$ , can be expressed as

$$W_f = (1/L) [W l_2 + h ((W/g) a - R_a \pm W \sin \theta_s)] \quad (2)$$

and

$$W_r = (1/L) [W l_1 - h ((W/g) a - R_a \pm W \sin \theta_s)] \quad (3)$$

where  $a$  is the deceleration. When the vehicle is moving uphill, the negative sign for the term  $W \sin \theta_s$  should be used. In the above expression, it is assumed that the aerodynamic resistance is applied at the center of gravity of the vehicle and that there is no drawbar load.

By considering the force equilibrium in the longitudinal direction, the following relationship can be established

$$F_b + f_r W = F_{bf} + F_{br} + f_r W = (W/g) a - R_a \pm W \sin \theta_s \quad (4)$$

where  $F_{bf}$  and  $F_{br}$  are the braking forces of the front and rear axles, respectively.

Substituting Eq. 4 into Eqs. 2 and 3 the normal loads on the axles become

$$W_f = (1/L) [W l_2 + h (F_b + f_r W)] \quad (5)$$

and

$$W_r = (1/L) [W l_1 - h (F_b + f_r W)] \quad (6)$$

The maximum braking force that the tire-ground contact can support is determined by the normal load and the coefficient of road adhesion. With four-wheel brakes, the maximum braking forces on the front and rear axles are given by (assuming the maximum braking force of the vehicle  $F_{bmax} = W$ ),

$$F_{bf max} = \mu W_f = (\mu W [ l_2 + h(\mu + f_r) ]) / L \quad (7)$$

$$F_{br max} = \mu W_r = (\mu W [ l_1 - h(\mu + f_r) ]) / L \quad (8)$$

where  $\mu$  is the coefficient of road adhesion. It should be noted that when the braking forces reach the values determined by Eqs. 7 and 8, tires are at the point of impending skid. Any variation would cause the tires to lock up.

It should be pointed out that the distribution of braking forces between the front and rear axles is a function of the design of brake system when no wheels are locked. For conventional brake systems, the distribution of braking forces is primarily dependent on the hydraulic (or pneumatic) pressures and brake cylinder (or chamber) areas in the front and rear brakes. From Eqs. 7 and 8 it can be seen that only when the distribution of braking forces between the front and rear axles is in exactly the same proportion as that of normal loads on the front and rear axles, will the maximum braking forces of the front and rear tires be developed at the same time.

$$K_{bf} / K_{br} = F_{bf max} / F_{br max} = (l_2 + h(\mu + f_r)) / (l_1 - h(\mu + f_r)) \quad (9)$$

where  $K_{bf}$  and  $K_{br}$  are the proportions of the total braking force on the front and rear axle, respectively, and are determined by the brake system design.

For instance, for a light truck with 68% of the static load on the rear axle ( $l_2/L = 0.32$ ,  $l_1/L = 0.68$ ),  $h/L = 0.18$ ,  $\mu = 0.85$  and  $f_r = 0.01$ , the maximum braking forces of the front and rear tires that the tire-ground contact can support will be developed at the same time, only if the braking force distribution between the front and rear brakes satisfies the following requirement

$$K_{bf} / K_{br} = (0.32 + 0.18 (0.85 + 0.01)) / (0.68 - 0.18 (0.85 + 0.01)) = 47 / 53$$

In other words, 47% of the total braking force must be placed on the front axle and 53% on the rear axle to achieve the optimum utilization of the potential braking capability of the vehicle. The braking force distribution that can ensure the maximum braking forces of the front and rear tires developed at the same time is referred to as the ideal braking force distribution. If the braking force distribution is not ideal, then either the front or the rear tires will lock up first.

When the rear wheels lock up first, the vehicle will lose directional stability. This can be visualized with the aid of next figure. The figure shows the top view of a two-axle vehicle acted upon by the braking force and the inertia force. When the rear tires lock, the capability of the rear tires to resist lateral force is reduced to zero. If some slight lateral movement of the rear tires is initiated by side wind, road camber, or centrifugal force, yawing moment due to the inertia force about the yaw center of the front axis will be developed. As the yaw motion progresses the moment arm of the inertia force increases, resulting in an increase in yaw acceleration. As the rear end of the vehicle swings around  $90^\circ$ , the moment arm gradually decreases and eventually the vehicle rotates  $180^\circ$  with the rear end leading the front end.

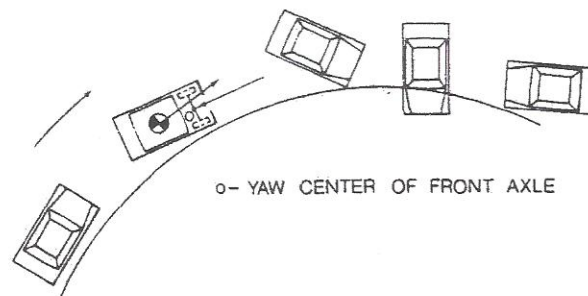


Fig. Loss of directional stability due to lockup of rear tires.

The lock up of front tires will cause a loss of directional control, and the driver will no longer be able to exercise effective steering. It should be pointed out, however, that front tires lockup does not cause directional instability. This is because whenever lateral movement of the front tires occurs, a self-correcting moment due to the inertia force of the vehicle about the yaw center of the rear axle will be developed. Consequently, it tends to bring the vehicle back to a straight line path.

Loss of steering control may be detected more readily by the driver and control may be regained by release or partial release of the brakes. Contrary to the case of front wheel lockup, when rear tires lock and the angular deviation of the vehicle exceeds a certain level, control cannot be regained even by complete release of the brakes and by the most skilful driving. This suggests that rear wheel lockup is more critical situation, particularly on a road surface with low coefficient of adhesion. Since on slippery surfaces, the value of the braking force is low, the kinetic energy of the vehicle will dissipate at a slow rate and the vehicle will experience a serious loss of directional stability over a considerable distance. Because of the importance of the sequence of locking of the tires to vehicle behavior during braking, it is necessary to determine quantitatively the conditions under which the rear tires will lock first;

To facilitate the understanding of the problem, only the braking force and rolling resistance will be considered in the following analysis. Thus

$$F_b + f_r W = F_{bf} + F_{br} + f_r W = (W/g) a \quad (10)$$

Substituting Eq. 10 into Eqs. 5 and 6 yields

$$W_f = (W/L)[l_2 + (a/g) h] \quad (11)$$

and

$$W_r = (W/L) [l_1 - (a/g) h] \quad (12)$$

The braking forces of the front and rear axles as determined by the brake system design are primarily related to the front and rear brake cylinder (or chamber) areas and are expressed by

$$F_{bf} = K_{bf} F_b = K_{bf} W [(a/g) - f_r] \quad (13)$$

and

$$F_{br} = K_{br} F_b = (1 - K_{bf}) F_b = (1 - K_{bf}) W [(a/g) - f_r] \quad (14)$$

The front tires approach impending lockup when

$$F_{bf} = \mu W_f \quad (15)$$

Substituting Eq. 11 and 13 into Eq. 5 yields

$$K_{bf} W [(a/g) - f_r] = \mu W [(l_2/L) + (a/g) (h/L)] \quad (16)$$

From Eq. 16, the vehicle deceleration rate (in g units) associated with the impending lockup of the front wheels can be defined by

$$(a/g)_f = [(\mu l_2/L) + K_{bf} f_r] / [K_{bf} - \mu h/L] \quad (17)$$

Similarly it can be shown that the rear tires approach impending lockup when the deceleration rate is

$$(a/g)_r = [(\mu l_1/L) + (1 - K_{bf}) f_r] / [(1 - K_{bf}) + \mu h/L] \quad (18)$$

For a given vehicle with a particular braking force distribution on a given road surface, the front tires lock first, if

$$(a/g)_f < (a/g)_r \quad (19)$$

On the other hand, the rear wheels will lock first if

$$(a/g)_r < (a/g)_f \quad (20)$$

From the above analysis, it can readily be seen that for a given vehicle with a fixed braking force distribution, both front and rear tires will lock at the same deceleration rate only on a particular road surface. Under this condition, the maximum braking forces of the front and rear axles that the tire-ground contact can support are developed at the same time, which indicates an optimum utilization of the potential braking capability of the vehicle. Under all other conditions, either the front or the rear wheels will lock first, resulting in loss of either steering control or directional stability. This suggests that ideally the braking force

distribution should be adjustable to ensure optimum braking performance under various operating conditions.

Based on the analysis described above, the interrelationships between the sequence of locking of wheels, the deceleration achievable prior to any wheel lockup, design parameters of the vehicle, and operating conditions can be quantitatively defined. As an illustrative example, Fig. 3.28 shows the braking characteristics of a light truck as a function of the braking effort distribution on the front axle under loaded and unloaded conditions. For the loaded condition, the gross vehicle weight is 44.48 kN (10,000 lb) and for the unloaded case it is 26.69 kN (6000 lb). The ratio of the height of the center of gravity to the wheelbase is 0.18 for both loaded and unloaded conditions. The coefficient of road adhesion is 0.85.

In Fig. 3.28, the solid line and the dotted line represent the boundaries of the deceleration rate that the vehicle can achieve prior to the locking of any wheels under loaded and unloaded conditions, respectively. Lines OA and O'A' represent the limiting values of deceleration rate the vehicle can achieve without locking the rear wheels, whereas line AB and A'B' represent the limiting values of deceleration rate the vehicle can achieve without locking the front tires. Use can be made of Fig. 3.28 to determine the braking characteristics of the light truck under various operating conditions. For instance, if the brake system is designed to have 40% of the total braking force placed on the front axle, then for the loaded vehicle, the lockup of the rear tires will take place prior to the lockup of the front tires and the highest deceleration rate the vehicle can achieve just prior to rear tires lockup will be 0.75 g. Conversely, if 60% of the total braking force is placed on the front, then for the loaded case the lockup of the front tires will take place prior to that of the rear tires, and the highest deceleration rate the vehicle can achieve without locking of any wheels will be 0.6 g. It is interesting to note that to achieve the maximum deceleration rate of 0.85 g, which indicates the optimum utilization of the potential braking capability on a surface with a coefficient of road adhesion of 0.85, 47% of the total braking force on the front is required for the loaded case as compared to 72% for the unloaded case. Therefore, there is a difference of 25% in the optimum braking force distribution between the loaded and unloaded cases. A compromise in the selection of the braking force distribution has to be made. Usually the value of the braking force distribution on the front axle corresponding to the intersection of lines AB and O'A', point 1 in Fig. 3.28 is selected. Under these circumstances, the maximum deceleration that the truck can achieve without locking any wheels under both loaded and unloaded conditions is 0.64 g on a surface with a coefficient of road adhesion of 0.85.

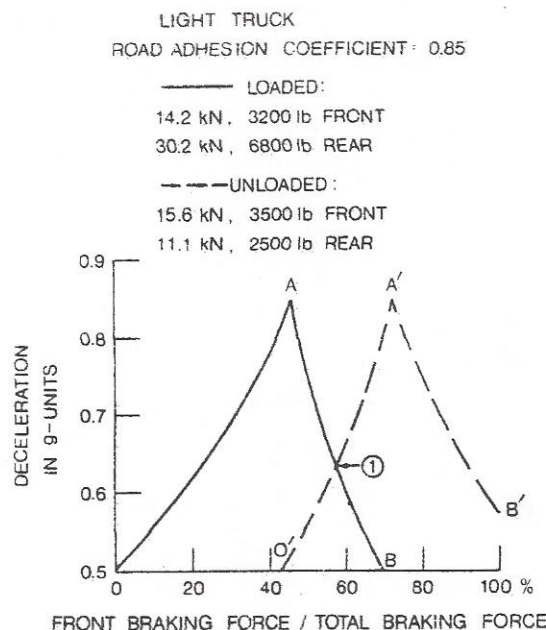


Fig. Effect of braking effort distribution on the braking performance of a light truck.

Figure 3.29 illustrates the braking characteristics of a passenger car. Because the difference in vehicle weight between rile loaded and unloaded cases for a passenger car is much smaller than that for a truck, the braking characteristics under these two conditions are very close, which car readily be seen from Fig. 3.29. to achieve the maximum deceleration rate of 0.85 g, 62% of the total braking force on the front is required for the loaded case as compared to 67% for the unloaded case, a difference of 5%. A braking force distribution with 64.5% of the total braking force on the front, corresponding to point 1 in Fig. 3.29, may be selected as a compromise under these circumstances. The maximum deceleration that the vehicle can achieve prior to any wheel lockup under both loaded and unloaded conditions is therefore 0.82 g.

The analysis and examples given above indicate the complex nature of the braking process. It is shown that the optimum braking force distribution, which ensures the maximum deceleration rate, varies with loading conditions of the vehicle, vehicle design parameters, and road surface conditions. In practice, the operating conditions vary in a wide range, thus for a given vehicle with a fixed braking force distribution, only under a specific set of loading and road conditions, will the maximum braking forces on the front and rear axles be developed at the same time and will the maximum deceleration rate be achieved. Under ail other conditions. the achievable deceleration rate without causing loss of steering control or directional stability will be reduced. To improve the braking performance and to ensure steering control and directional stability under all possible operating conditions, antilock devices and the like have been introduced. The prime function of these devices is to prevent wheels from locking, thus the capability of the wheels to sustain side force can be maintained. The operating principles of the antilock system will be briefly described in next section.

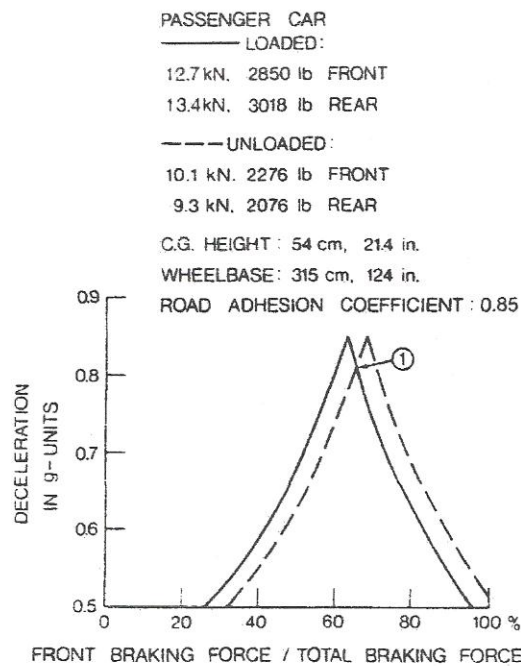


Fig. Effect of braking effort distribution on the braking performance of a passenger car. (Reproduced by permission of the Society of Automotive Engineers from reference 3.12.)

### Example

A passenger car weighs 21.24 kN (4775 lb) and has a wheelbase of 2.87 m (113 in.). The center of gravity is 1.27 m (50 in.) behind the front axle and 0.508 m (20 in.) above ground level. The braking effort distribution on the front wheels is 60%. The coefficient of rolling

resistance is 0.02. Determine which set of the wheels will lock first on two road surfaces: one with a coefficient of adhesion  $\mu = 0.8$  and the other with  $\mu = 0.2$ .

**Solution,**

**A** On the road surface with  $\mu=0.8$ , the vehicle deceleration associate with the impending lockup of the front wheels is determined by Eq.17

$$(a/g)_f = [(\mu l_2/L) + K_{bf} f_r] / [K_{bf} - \mu h/L]$$

$$= (0.8 \times 0.558 + 0.6 \times 0.02) / (0.6 - 0.8 \times 0.177) = 1.0$$

The vehicle deceleration associated with the impending lockup of the rear wheels is determined by Eq.18

$$(a/g)_r = [(\mu l_1/L) + (1 - K_{bf}) f_r] / [(1-K_{bf}) + \mu h/L]$$

$$= (0.8 \times 0.442 + 0.4 \times 0.02) / (0.4 + 0.8 \times 0.177) = 0.67$$

Since  $(a/g)_f > (a/g)_r$ , the rear wheels will lock first on the road surface with  $\mu = 0.8$ .

**B** On the road surface with  $\mu = 0.2$

$$(a/g)_f = (0.2 \times 0.558 + 0.6 \times 0.02) / (0.6 - 0.2 \times 0.177) = 0.219$$

$$(a/g)_r = (0.2 \times 0.442 + 0.4 \times 0.02) / (0.4 + 0.2 \times 0.177) = 0.221$$

Since  $(a/g)_f < (a/g)_r$ , the front wheels will lock first on the road surface with  $\mu = 0.2$ .

### Braking Efficiency and Stopping Distance

To characterize the braking performance of a road vehicle, braking efficiency, may be used. Braking efficiency  $\eta_b$  is defined as the ratio of the maximum deceleration rate in g units ( $a/g$ ) achievable prior to any wheel lockup to the coefficient of road adhesion  $\mu$ , and is given by:

$$\eta_b = (a/g) / \mu \tag{21}$$

The braking efficiency indicates the extent to which the vehicle utilizes the coefficient of road adhesion available during braking. Thus, when  $a/g < \mu$  hence  $\eta_b < 1$ , the deceleration is less than the maximum achievable, resulting in unnecessarily long stopping distance. Referring to Fig 3.28, if 57% of the total braking force is placed on the front, corresponding to point 1, the maximum deceleration achievable prior to any wheel lockup is 0.64 g. This indicates that on a surface with a coefficient of road adhesion of 0.85, the braking efficiency is 75.3%.

Stopping distance is another parameter widely used for evaluating the overall braking performance of road vehicle. To predict the stopping distance, the basic principles in Dynamics are employed. The interrelationships between stopping distance, braking force, vehicle mass, and vehicle speed, in differential form, may be expressed as:

$$Ads = ((F_b + \Sigma R) / (\gamma_b W / g)) ds = V dV \quad (22)$$

where  $\gamma_b$  is an equivalent mass factor taking into account the mass moments of inertia of the rotating components involved during braking. Since during braking clutch is usually disengaged, the value of  $\gamma_b$  is not necessarily the same as that of  $\gamma_m$  used in the calculation of acceleration. For conventional automotive vehicles,  $\gamma_b$  may be taken as approximately 1.04.

Equation 22 may be integrated to determine the stopping distance  $S$  from an initial speed  $V_1$  to a final speed  $V_2$

$$S = \int_{V_2}^{V_1} \frac{\gamma_b W}{g} \frac{V dV}{F_b + \Sigma R} \quad (23)$$

Substituting Eq. 1 into the above equation and neglecting the transmission resistance  $R_t$ , Eq. 23 becomes

$$S = \frac{\gamma_b W}{g} \int_{V_2}^{V_1} \frac{V dV}{F_b + f_r W \cos \theta_s \pm W \sin \theta_s + R_a} \quad (24)$$

The aerodynamic resistance is proportional to the square of speed and it may be expressed as

$$R_a = \frac{\rho}{2} C_D A_f V^2 = C_{ae} V^2 \quad (25)$$

With substitution of  $C_{ae}$  for  $R_a$  and integration, stopping distance can be expressed by

$$S = \frac{\gamma_b W}{2gC_{ae}} \ln \left( \frac{F_b + f_r W \cos \theta_s \pm W \sin \theta_s + C_{ae} V_1^2}{F_b + f_r W \cos \theta_s \pm W \sin \theta_s + C_{ae} V_2^2} \right) \quad (26)$$

For final speed  $V_2 = 0$ , Eq.26 reduces to the form

$$S = \frac{\gamma_b W}{2gC_{ae}} \ln \left( 1 + \frac{C_{ae} V_1^2}{F_b + f_r W \cos \theta_s \pm W \sin \theta_s} \right) \quad (27)$$

For a given vehicle, if the braking force distribution and road conditions are such that the maximum braking Forces of the front and rear wheels that the tire-ground contact can support are developed at the same time, that is braking efficiency  $\eta_b = 100\%$  the minimum stopping distance will be achieved. In this case, the braking torque generated by the brakes have already overcome the inertia of the rotating parts connected with the wheels, the maximum braking forces developed at the tire-ground contact are retarding only the translational inertia. The mass factor  $\gamma_b$  is therefore one. The minimum stopping distance  $S_{min}$  can be expressed as,

$$S_{min} = \frac{W}{2gC_{ae}} \ln \left( 1 + \frac{C_{ae} V_1^2}{\mu W + f_r W \cos \theta_s \pm W \sin \theta_s} \right) \quad (28)$$

If the braking efficiency  $\eta_b$ , is less than 100% (i.e., the maximum deceleration rate in g units achievable prior to wheel lockup is less than the coefficient of road adhesion available) then the stopping distance will be longer than that determined using Eq. 3.68. In this case, the stopping distance may be calculated from

$$S = \frac{W}{2gC_{ae}} \ln \left( 1 + \frac{C_{ae} V_1^2}{\eta_b \mu W + f_r W \cos \theta_s \pm W \sin \theta_s} \right) \quad (29)$$



It should be pointed out that in practice there is a time lag between the application of brakes and the full development of the braking force. This time lag depends on the response of the brake system. The actual stopping distance therefore will be longer than that calculated using the equations given above. In general, the distance the vehicle travels during the transient period between the application of brake and the attainment of steady-state braking has to be taken into consideration in determining the total stopping distance. In a first approximation, this additional stopping distance  $S_d$ , may be calculated from

$$S_d = t_d V_1 \quad (30)$$

where  $t_d$  is the response time of the brake system and  $V_1$  the initial speed of the vehicle. For preliminary braking performance calculations, an average value of 0.3 s for  $t_d$  may be assumed. The delay in applying brakes due to driver's reaction time further increases the actual stopping distance in practice. This reaction time usually varies from 0.5 to 2 s.

### Antilock Brake Systems

As mentioned previously, when a tire is locked (i.e., 100% skid), the coefficient of road adhesion fails to its sliding value and its ability to sustain side force is reduced to almost null. As a result, the vehicle will lose directional control or stability, and the stopping distance will be longer than the minimum achievable. Figure 332 shows the general characteristics of the coefficient of braking effort  $\mu$  and the coefficient of cornering force  $\mu_c$  at a given slip angle, which is the ratio of cornering force to vertical load, as a function of skid for a pneumatic tire.

The prime function of an antilock system is to prevent the tire from locking and to keep the skid of the tire within a desired range such as that shown in Fig. 3.32. This will ensure that the tire can develop sufficiently high braking force for stopping the vehicle and at the same time can provide adequate cornering force for directional control and stability.

A modern antilock system usually consists of a sensor, a control unit, and a brake pressure modulator as shown in Fig. 3.33. In practice, the skid of a tire is difficult to measure directly, therefore, the control logic of the system is usually formulated based on some easily measurable parameters, such as the angular speed and/or the angular acceleration of the wheel.

The sensor is to monitor the parameters specified and to generate signals representative of those parameters. When the angular speed or angular acceleration of the wheel is used as the basic parameter, the sensor is mounted on the wheel. The signals generated by the sensor are transmitted to the control unit.

The control unit usually consists of four modules: a signal processing module, a module for predicting whether the wheel is at the point of locking, a module for determining whether the danger of locking the wheel is averted, and a module for generating a command signal for activating the pressure modulator.

In the control unit, after the signals generated by the sensor have been processed, the measured parameters and/or those derived from them are compared with the corresponding predetermined threshold values. When certain requirements that indicate the impending lockup of the wheel are met by the measured parameters and/or their derivatives, a command signal is sent to the modulator to release the brake. The methods for predicting the locking of the wheel used in some existing antilock systems are described below.

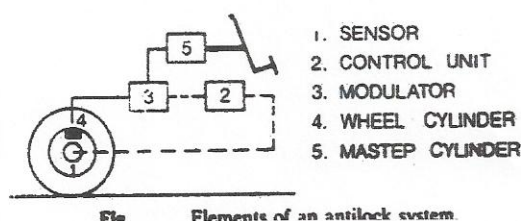


Fig. Elements of an antilock system.

**A** In some of the existing antilock devices, the locking of the wheel is predicted and a command signal is sent to the modulator to release the brake, whenever the product of the angular deceleration of the wheel and its rolling radius exceeds a predetermined value. In some systems, the threshold value used is in the range 1.5-2.5 g.

**B** In an antilock system designed for passenger cars, a signal proportional to the angular speed of the wheel is tracked by a track and hold circuit in the control unit, and when the equivalent linear deceleration of the center of the wheel is greater than 1.6 g, the tracked signal is held in a memory circuit for about 140 ms. During this period of time, if the actual angular speed of the wheel decreases by 5% of the already held value and at the same time if the deceleration of the vehicle measured is not higher than 0.5 g, it is predicted that the wheel is at the point of locking, and a command signal for releasing the brake is sent to the modulator. On the other hand, if the deceleration of the vehicle is higher than 0.5 g, locking of the wheel is predicted and the brake is released whenever the decrease in angular speed of the wheel is 15% of the already stored value.

During the braking process, the operating conditions of the wheel and of the vehicle are continuously monitored by the sensor and the control unit. After the danger of locking the wheel is predicted and the brake is released, another module in the control unit will determine at what point the brake should be reapplied. There are a variety of criteria employed in existing antilock systems; some of them are described below.

**A** In some systems, a command signal will be sent to the modulator to reapply the brake, whenever the criteria for releasing the brake discussed previously are no longer satisfied.

**B** In certain devices, a fixed time delay is introduced to ensure that the brake is reapplied only when a fixed time has elapsed after the release of the brake.

**C** In some systems, the brake is reapplied as soon as the product of the angular acceleration of the wheel and the rolling radius exceeds a predetermined value, after the brake is released. Threshold values of linear acceleration in the range 3.06.0 g have been used.